Ausführliche Lösungen

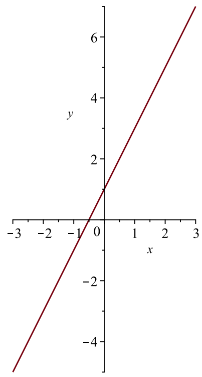
12.1 Die Graphen haben von oben links nach unten rechts folgende Gleichungen:

$$y = -\frac{1}{2}x + 1 \qquad y = x^2 - 1$$
$$y = \sin(x) \qquad y = \ln(x)$$
$$y = e^x \qquad y = \tan(x)$$

◀

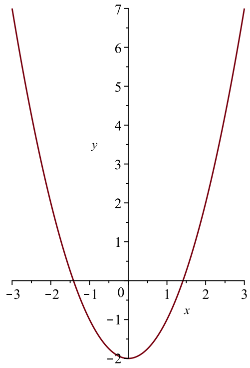
12.2

$x \mapsto 2x + 1$



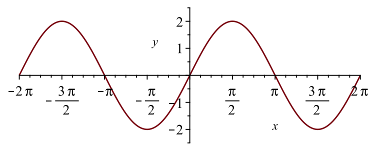
Definitionsbereich $\mathbb{R}$, Wertebereich $\mathbb{R}$ ◀

$x \mapsto x^2 - 2$



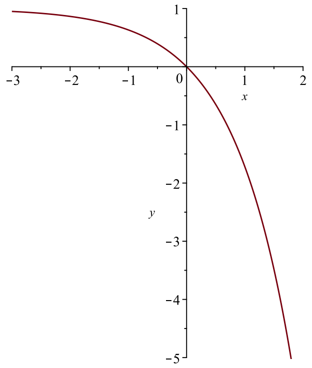
Definitionsbereich $\mathbb{R}$, Wertebereich $[-2; \infty[$ ◀

$$x \mapsto 2\sin(x)$$



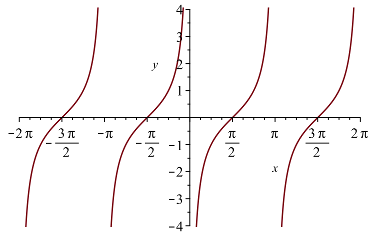
Definitionsbereich $\mathbb{R}$, Wertebereich $[-2; 2]$ ◀

$$x \mapsto -e^x + 1$$



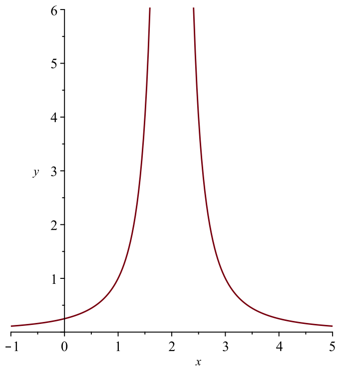
Definitionsbereich $\mathbb{R}$, Wertebereich $]-\infty; 1[$ ◀

$$x \mapsto \tan\left(x + \frac{\pi}{2}\right)$$



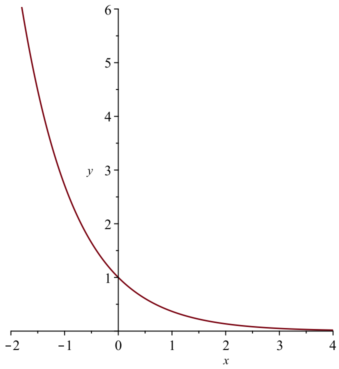
Definitionsbereich $\{x \in \mathbb{R} \mid x \neq k\pi,\ k \in \mathbb{Z}\}$, Wertebereich $\mathbb{R}$ ◀

$$x \mapsto \frac{1}{(x-2)^2}$$



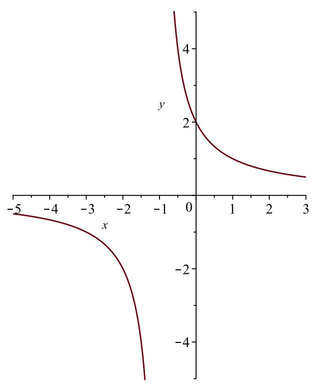
Definitionsbereich $\mathbb{R}\backslash\{2\}$, Wertebereich $]0;\infty[$ ◀

$$x \mapsto e^{-x}$$



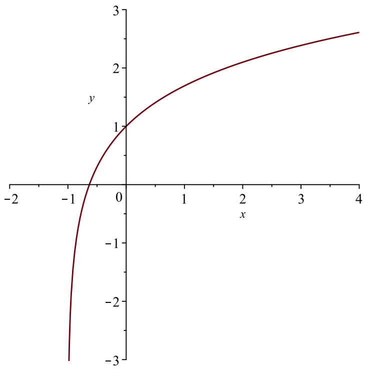
Definitionsbereich $\mathbb{R}$, Wertebereich $]0;\infty[$ ◀

$$x \mapsto \frac{2}{x+1}$$



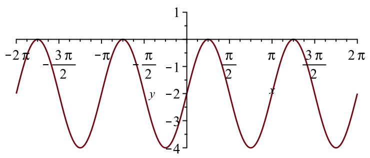
Definitionsbereich $\mathbb{R}\backslash\{-1\}$, Wertebereich $\mathbb{R}\backslash\{0\}$ ◀

$$x \mapsto \ln(1+x)+1$$



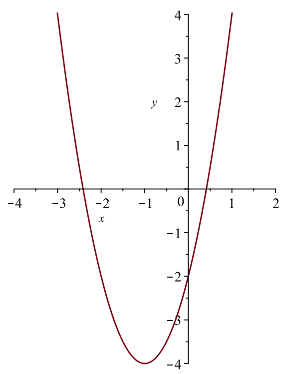
Definitionsbereich $]-1;\infty[$, Wertebereich $\mathbb{R}$ ◀

$$x \mapsto 2\sin(2x)-2$$



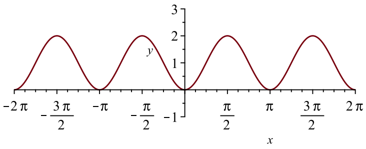
Definitionsbereich $\mathbb{R}$, Wertebereich $[-4;0]$ ◀

$$x \mapsto 2(x+1)^2-4$$



Definitionsbereich $\mathbb{R}$, Wertebereich $[-4;\infty[$ ◀

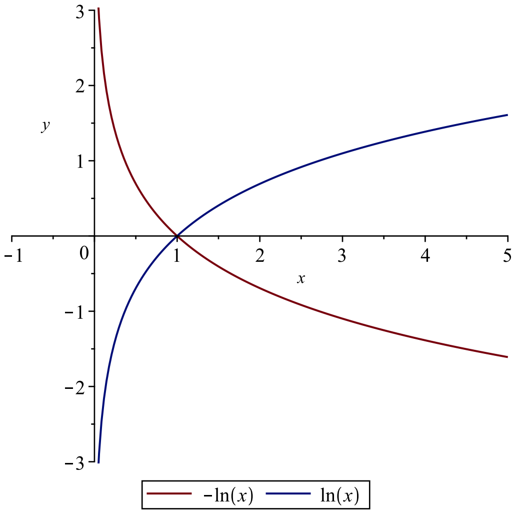
$$x \mapsto -\cos\left(2(x-\pi)\right)+1$$



Definitionsbereich $\mathbb{R}$, Wertebereich $[0;2]$ ◀

12.3

a) $y = -\ln(x)$



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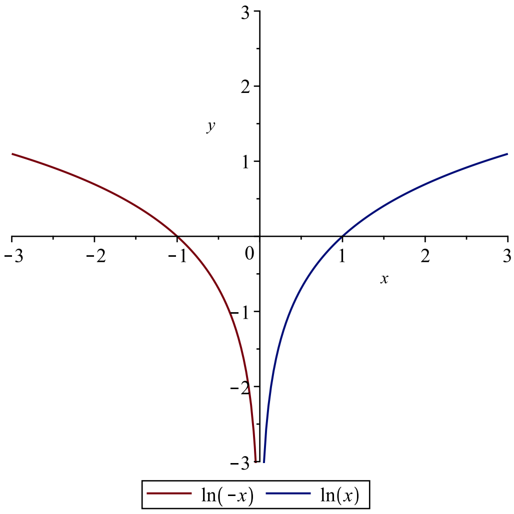
b) $y = \ln(-x)$



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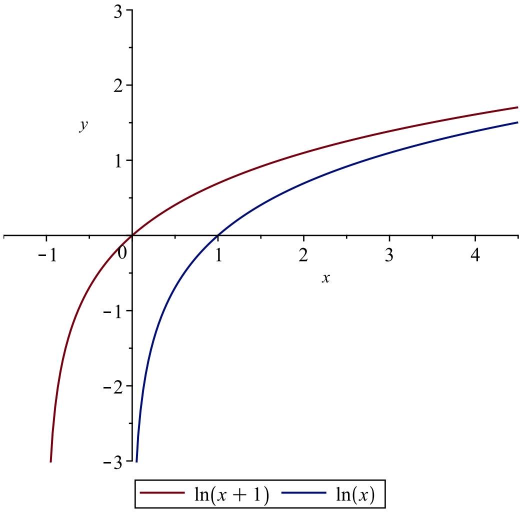
c) $y = \ln(x+1)$



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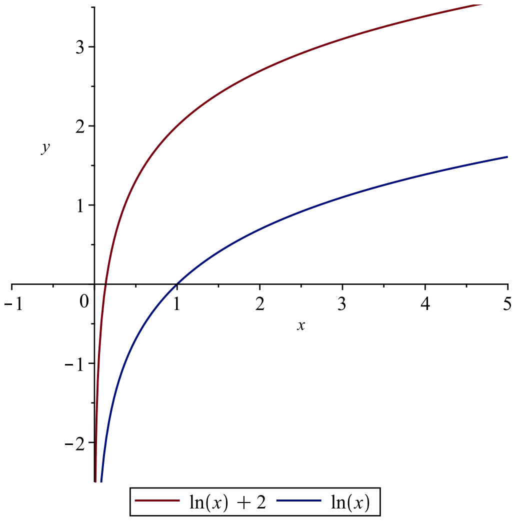
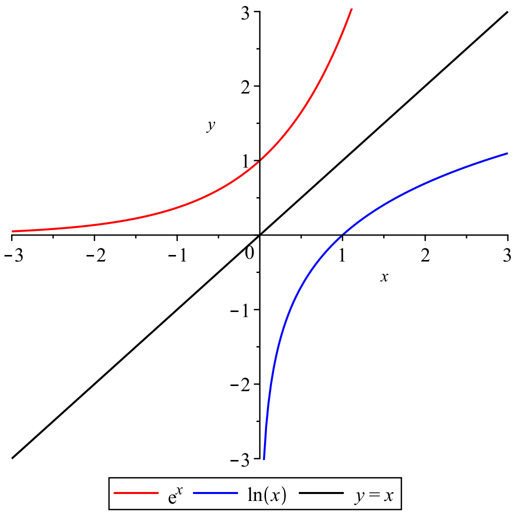
d) $y = \ln(x) + 2$



e) $y = e^x$



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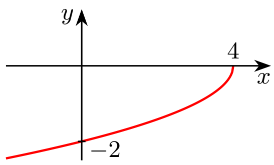
12.4 Umwandlung der Funktionsterme, sodass man die Transformationen ablesen kann.

$$y = -\sqrt{4-x} = -\sqrt{(-1)(x-4)}$$



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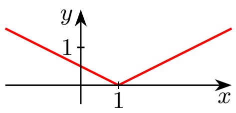
$$y = \frac{|x-1|}{2} = \begin{cases} \frac{1}{2}(x-1) & \text{für} \quad x \geq 1 \\ \frac{1}{2}(-x+1) & \text{für} \quad x < 1 \end{cases}$$



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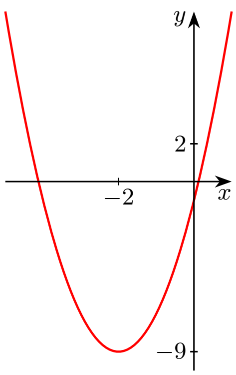
$$y = 2x^2 + 8x - 1 = 2(x+2)^2 - 9$$



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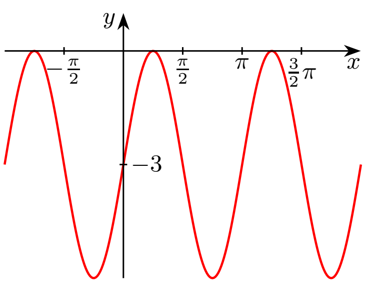
$$y = 3\left(\sin\left(-2x - \pi\right) - 1\right) = 3\sin\left((-2)\left(x + \frac{\pi}{2}\right)\right) - 3$$



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12.5

a) Die Aussage ist falsch, z. B. hat $y = x^2 - 1$ gleich zwei Nullstellen; ◀

b) Die Aussage ist wahr, da die Funktion für negative x gegen $-\infty$ und für positive x gegen $+\infty$ strebt oder umgekehrt; ◀

c) Die Aussage ist wahr, da eine Parabel entweder unten oder oben einen Scheitel und damit ein Extremum hat; ◀

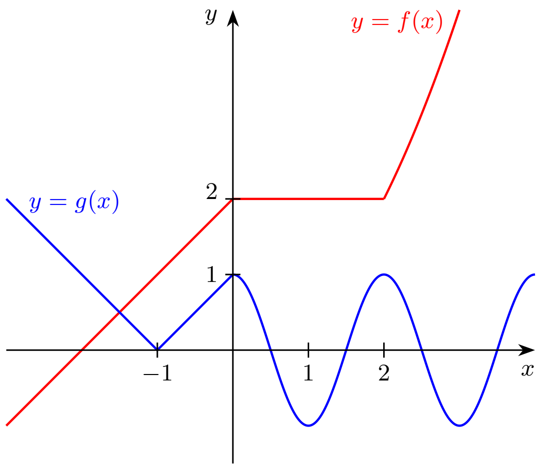
d) Die Aussage ist falsch, da $x = 0$ eine Definitionslücke ist; ◀

e) Die Aussage ist falsch, da durch die Funktion z. B. der Wert $y = 0$ nicht erreicht wird; ◀

f) Die Aussage ist falsch, Gegenbeispiel $f(x) = \frac{x}{1+x^2}$; ◀

g) Die Aussage ist wahr, die Wendestellen $x = k\pi$ $(k \in \mathbb{Z})$ der Sinusfunktion sind die Maximal- bzw. Minimalstellen der Kosinusfunktion. ◀

12.6



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12.7

a) $(f + g)(x) = (x - 2) + (1 - 2x) = -x - 1$
$D_{f+g} = \mathbb{R}$

$(f - g)(x) = (x - 2) - (1 - 2x) = x - 2 - 1 + 2x = 3x - 3$
$D_{f-g} = \mathbb{R}$

$(f \cdot g)(x) = (x - 2) \cdot (1 - 2x) = x - 2x^2 - 2 + 4x = -2x^2 + 5x - 2$
$D_{f\cdot g} = \mathbb{R}$

$\frac{f}{g}(x) = \frac{x-2}{1-2x}$
$D_{\frac{f}{g}} = \mathbb{R}\backslash\{\frac{1}{2}\}$ ◀

b) $(f + g)(x) = \sin(x) + \cos(x)$
$D_{f+g} = \mathbb{R}$

$(f - g)(x) = \sin(x) - \cos(x)$
$D_{f-g} = \mathbb{R}$

$(f \cdot g)(x) = \sin(x) \cdot \cos(x)$
$D_{f\cdot g} = \mathbb{R}$

$\frac{f}{g}(x) = \frac{\sin(x)}{\cos(x)} = \tan(x)$
$D_{\frac{f}{g}} = \left\{x \in \mathbb{R} \mid x \neq \frac{\pi}{2} + k\pi, k \in \mathbb{Z}\right\}$ ◀

c) $(f+g)(x) = (x^2-1)+(x-1) = x^2+x-2$
$D_{f+g} = \mathbb{R}$
$(f-g)(x) = (x^2-1)-(x-1) = x^2-1-x+1 = x^2-x$
$D_{f-g} = \mathbb{R}$
$(f\cdot g)(x) = (x^2-1)\cdot(x-1) = x^3-x^2-x+1$
$D_{f\cdot g} = \mathbb{R}$
$\frac{f}{g}(x) = \frac{x^2-1}{x-1} = \frac{(x+1)(x-1)}{x-1} = x+1$
$D_{\frac{f}{g}} = \mathbb{R}\backslash\{1\}$ ◀

d) $(f+g)(x) = \sqrt{1+x}+\sqrt{x}$
$D_{f+g} = [0;\infty[$
$(f-g)(x) = \sqrt{1+x}-\sqrt{x}$
$D_{f-g} = [0;\infty[$
$(f\cdot g)(x) = \sqrt{1+x}\cdot\sqrt{x} = \sqrt{(1+x)x} = \sqrt{x+x^2}$
$D_{f\cdot g} = [0;\infty[$
$\frac{f}{g}(x) = \frac{\sqrt{1+x}}{\sqrt{x}} = \sqrt{\frac{1+x}{x}}$
$D_{\frac{f}{g}} =]0;\infty[$ ◀

12.8

a) $f(x) = x+1, \quad g(x) = \ln(x)$ ◀

b) $f(x) = \frac{x+2}{x+1}, \quad g(x) = x^2$ ◀

c) $f(x) = \cos(x), \quad g(x) = x^2$ ◀

12.9

a) $(g\circ f)(x) = g(f(x)) = \sqrt{x^2} = |x|, \quad D_{g\circ f} = \mathbb{R}$
$(f\circ g)(x) = f(g(x)) = \sqrt{x}^2 = x, \quad D_{f\circ g} = [0;\infty[$ ◀

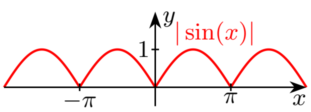
b) $(f\circ(g+h))(x) = f(g(x)+h(x)) = \left(\sqrt{x}+\frac{1}{x}\right)^2 = x+\frac{2}{\sqrt{x}}+\frac{1}{x^2}, \quad D_{f\circ(g+h)} =]0;\infty[$ ◀

c) $(h\circ(f\cdot g))(x) = h(f(x)\cdot g(x)) = \frac{1}{x^2\cdot\sqrt{x}} = \frac{1}{x^{\frac{5}{2}}}, \quad D_{f\circ(g\cdot h)} =]0;\infty[$ ◀

d) $\left(f\circ\left(\frac{g}{h}\right)\right)(x) = f\left(\frac{g(x)}{h(x)}\right) = (\sqrt{x}\cdot x)^2 = x^3, \quad D_{f\circ\left(\frac{g}{h}\right)} =]0;\infty[$ ◀

e) $(f\circ(g\circ h))(x) = \sqrt{\frac{1}{x}}^{-2} = f(g(h(x))) = \frac{1}{x}, \quad D_{f\circ(g\circ h)} =]0;\infty[$ ◀

12.10



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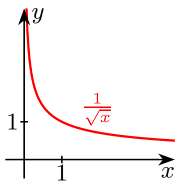

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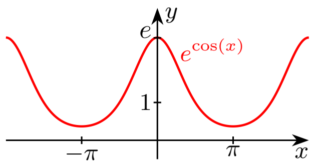

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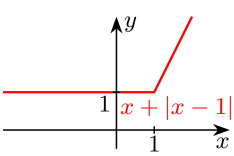

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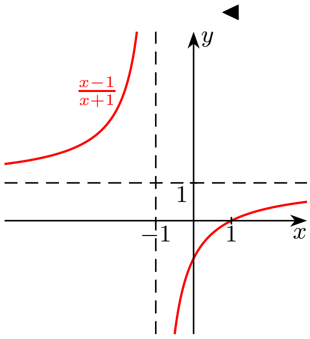

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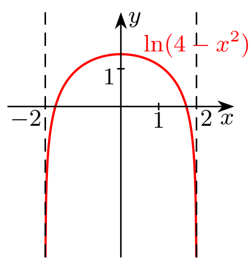

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12.11

a) Ansatz:

$$f(x) = a_2x^2 + a_1x + a_0$$

Einsetzen der bekannten Werte:

$$\begin{array}{lccccccc}
P_0(0|-3): & & & & & a_0 & = & -3 \\
P_1(1|0): & a_2 & + & a_1 & + & a_0 & = & 0 \\
P_2(2|5): & 4a_2 & + & 2a_1 & + & a_0 & = & 5
\end{array}$$

Als Lösung dieses LGS ergibt sich

$$a_2 = 1, \qquad a_1 = 2, \qquad a_0 = -3$$

und damit als Funktionsterm

$$f(x) = x^2 + 2x - 3.$$

b) Ansatz:

$$f(x) = a_3x^3 + a_2x^2 + a_1x + a_0$$

Einsetzen der bekannten Werte:

$$\begin{array}{lcccccccc}
P_0(-1|-17): & -a_3 & + & a_2 & - & a_1 & + & a_0 & = & -17 \\
P_1(0|-5): & & & & & & & a_0 & = & -5 \\
P_2(1|1): & a_3 & + & a_2 & + & a_1 & + & a_0 & = & 1 \\
P_3(2|13): & 8a_3 & + & 4a_2 & + & 2a_1 & + & a_0 & = & 13
\end{array}$$

Als Lösung dieses LGS ergibt sich

$$a_3 = 2, \qquad a_2 = -3, \qquad a_1 = 7, \qquad a_0 = -5$$

und damit als Funktionsterm

$$f(x) = 2x^3 - 3x^2 + 7x - 5.$$

12.12 Ansatz:

$$g(x) = a_2x^2 + a_1x + a_0$$

Multiplikation mit $(x-2)^2$:

$$\begin{aligned}
(x^2-2)\cdot g(x) &= (x^2-2)(a_2x^2 + a_1x + a_0) \\
&= a_2x^4 + a_1x^3 + a_0x^2 - 2a_2x^2 - 2a_1x - 2a_0 \\
&= a_2x^4 + a_1x^3 + (a_0 - 2a_2)x^2 - 2a_1x - 2a_0
\end{aligned}$$

Koeffizientenvergleich in

$$a_2x^4 + a_1x^3 + (a_0 - 2a_2)x^2 - 2a_1x - 2a_0 = x^4 - 3x^3 + 6x - 4$$

ergibt

$$a_2 = 1, \qquad a_1 = -3, \qquad a_0 = 2,$$

also

$$g(x) = x^2 - 3x + 2.$$